then

$$\begin{aligned}&3\,|x+y+z|^2+|2x-y-z|^2+|2y-z-x|^2+|2z-x-y|^2\\&=3\left(|x|^2+|y|^2+|z|^2\right)+6\left(|x|^2+|y|^2+|z|^2\right)=9\left(|x|^2+|y|^2+|z|^2\right)\end{aligned}$$

and the original inequality becomes

$$81\left(\frac{1}{|x|^2}+\frac{1}{|y|^2}+\frac{1}{|z|^2}\right)^{-1}\le 9\left(|x|^2+|y|^2+|z|^2\right)$$

or

$$9\left(\frac{1}{|x|^2}+\frac{1}{|y|^2}+\frac{1}{|z|^2}\right)^{-1}\le \left(|x|^2+|y|^2+|z|^2\right),$$

where latter inequality holds because by Cauchy's inequality

$$\left(|x|^2+|y|^2+|z|^2\right)\left(\frac{1}{|x|^2}+\frac{1}{|y|^2}+\frac{1}{|z|^2}\right)\ge 9$$

**Also solved by José Gibergans-Báguena, BARCELONA TECH, Barcelona, Spain.**

**60.** *Compute the following sum:*

$$\sum_{1\le i_1\le n+1}\frac{1}{i_1}+\sum_{1\le i_1<i_2\le n+1}\frac{1}{i_1 i_2}+\ldots+\sum_{1\le i_1<\ldots<i_n\le n+1}\frac{1}{i_1 i_2\ldots i_n}$$

(Training Spanish Team for VJIMC 2014)

**Solution 1 by José Gibergans-Báguena, BARCELONA TECH, Barcelona, Spain.** Let us denote by $S$ the following sum:

$$\begin{aligned}S&=1+\sum_{1\le i_1\le n+1}\frac{1}{i_1}+\sum_{1\le i_1<i_2\le n+1}\frac{1}{i_1 i_2}+\ldots+\frac{1}{1\cdot 2\cdots n+1}\\&=\prod_{k=1}^{n+1}\left(1+\frac{1}{k}\right)=\prod_{k=1}^{n+1}\frac{k+1}{k}=\frac{(n+2)!}{(n+1)!}=n+2\end{aligned}$$

Then

$$\begin{aligned}&\sum_{1\le i_1\le n+1}\frac{1}{i_1}+\sum_{1\le i_1<i_2\le n+1}\frac{1}{i_1 i_2}+\ldots+\sum_{1\le i_1<\ldots<i_n\le n+1}\frac{1}{i_1 i_2\ldots i_n}\\&=S-\left(1+\frac{1}{1\cdot 2\cdots n+1}\right)=n+1-\frac{1}{(n+1)!}\end{aligned}$$

and we are done. □

**Solution 2 by Arkady Alt, San Jose, California, USA.** Consider the polynomial

$$P(x)=\prod_{k=1}^{n+1}\left(x+\frac{1}{k}\right)=x^{n+1}+a_{n+1}+\sum_{k=1}^{n}a_k x^{n+1-k}$$

Then by general Vieta's Theorem

$$a_k = \sum_{1 \le i_1 < i_2 < \ldots < i_k \le n+1} \frac{1}{i_1 i_2 \ldots i_k},\ 1 \le k \le n+1$$

Since $a_{n+1} = \dfrac{1}{(n+1)!}$ then original sum is $a_1 + a_2 + \ldots + a_n = P(1) - 1 - \dfrac{1}{(n+1)!}$. From the other hand

$$P(1) = \prod_{k=1}^{n+1} \left(1 + \frac{1}{k}\right) = \prod_{k=1}^{n+1} \frac{k+1}{k} = n+2$$

Hence, $a_1 + a_2 + \ldots + a_n = (n+2) - 1 - \dfrac{1}{(n+1)!} = n + 1 - \dfrac{1}{(n+1)!}$.

**Solution 3 by Omran Kouba, Higher Institute for Applied Sciences and Technology, Damascus, Syria.** Let $S_n$ be the proposed sum and let $\mathbb{N}_{n+1} = \{1, 2, \ldots, n+1\}$. We will use the following formula:

$$\prod_{k=1}^{n+1} (1 + b_k) = \sum_{k=0}^{n+1} \Big( \sum_{\substack{X \subset \mathbb{N}_{n+1} \\ |X| = k}} \prod_{j \in X} b_j \Big),$$

that can be proved by induction. Taking $b_j = 1/j$ and noting that terms corresponding to $X = \emptyset$ and $X = \mathbb{N}_{n+1}$ are missing in the proposed sum, we see that

$$S_n = \prod_{k=1}^{n+1} \left(1 + \frac{1}{k}\right) - 1 - \frac{1}{(n+1)!} = n + 1 - \frac{1}{(n+1)!}$$

**Also solved by Paolo Perfetti, Department of Mathematics, Tor Vergata University, Rome, Italy and José Luis Díaz-Barrero, BARCELONA TECH, Barcelona, Spain.**

**Editor's Comment:** On the last issue we forgot to acknowledge two more solutions of Problem 55 of the MathContest section we received. One from Angel Plaza, independently, and another from Yiri D. Valencia, Himar A. Fabelo (students), and Angel Plaza (jointly), University of Las Palmas, Gran Canaria, Spain.